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(I'VE TAKEN 1 WEEK BREAK FROM THIS)

The Safety of the Blood Supply — Time to Raise the Bar

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The Safety of the Blood Supply — Time to Raise the Bar

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Existing and emerging pathogens including viruses, bacteria, protozoa, and prions continue to threaten the safety of the blood supply. Blood-collecting facilities, including community and hospital blood banks, have typically relied on a reactive approach to these threats, developing and implementing screening tests after potential pathogens are identified. During this often slow and laborious process, pathogen transmission through transfusion is inevitable. The Food and Drug Administration (FDA) recently approved three new pathogen-reduction technologies (see table). These systems are capable of inactivating a wide variety of pathogens in

Pathogen-Reduction Technologies Approved and in Development in the United States and Europe.*

Component and Source	Manufacturer and Technology	Treatment Process	Manner of Inhibiting Replication	Regulatory Status
Platelets				
Individual volunteer donors	Cerus Intercept Blood System	Psoralen (amotosalen) and UVA light exposure	Formation of DNA and RNA monoadducts and cross-linkage	FDA approved; CE marked
	Terumo BCT Mirasol Pathogen Reduction Technology (PRT) System	Riboflavin and ultraviolet light exposure	Direct DNA and RNA damage and guanine modification	Phase 3 study planned in the United States; CE marked
	Macopharma Theraflex ultraviolet platelets	UVC light exposure	Direct DNA and RNA damage and thymine dimer formation	CE marked
Plasma				
Pools of volunteer and paid donors	Octapharma Octaplas	Plasma pools treated with solvent, tri-*n*-butyl phosphate and detergent (octoxynol)	Lipid membrane disruption of enveloped viruses	FDA approved; CE marked
Individual and minipools of volunteer donors	Cerus Intercept Blood System	Psoralen (amotosalen) and UVA light exposure	Formation of DNA and RNA monoadducts and cross-linkage	FDA approved; CE marked
Individual volunteer donors	Macopharma Theraflex MB Plasma System	Filtration, methylene blue treatment and visible light exposure	DNA and RNA damage by type I and type II redox reactions	CE marked
	Terumo BCT Mirasol PRT System	Riboflavin and ultraviolet light exposure	Direct DNA and RNA damage and guanine modification	CE marked
Whole blood				
Individual volunteer donors	Terumo BCT Mirasol PRT System	Riboflavin and ultraviolet light exposure	Direct DNA and RNA damage and guanine modification	Phase 3 studies planned in the United States, completed in Africa
Red cells				
Individual volunteer donors	Cerus Intercept Blood System	Frangible Anchor-Linker Effector (S303) and glutathione	Formation of DNA and RNA monoadducts and cross-linkage	U.S. phase 2 and European phase 3 studies complete

* The downsides of pathogen reduction vary by technology and include relative loss of component yield and reduced functionality, unknown residual infectivity of agents with pathogen loads that exceed validated inactivation efficacy, and resistance by certain pathogens (e.g., nonenveloped viruses, for certain technologies, and spore-forming bacteria). Short-term and long-term clinical adverse events have not been reproducibly documented. A listing of countries using each technology is available at www.aabb.org/tm/eid/Pages/pathogen-reduction-systems.aspx. CE (Conformité Européenne) denotes compliance with requirements in the European Union, FDA Food and Drug Administration, UVA ultraviolet A, and UVC ultraviolet C.

donated blood components, potentially eliminating threats — including some for which no other intervention exists. We believe the FDA should mandate a proactive approach, ensuring ongoing blood safety by requiring treatment of blood components by approved pathogen-reduction technologies.

One of the newly approved systems treats plasma by using a solvent and detergent to dissolve lipid membranes, thereby rendering pathogens noninfectious. Another technology, approved for treatment of plasma and platelets, uses a psoralen compound, amotosalen, which binds and cross-links nucleic acids when exposed to ultraviolet A (UVA) light. A third technology using ultraviolet light and riboflavin (vitamin B2) is being tested in trials. Numerous studies demonstrate little substantive negative effect from pathogen reduction on plasma proteins or platelets. Hemoglobin absorption of UVA light prevents treatment of red cells with current pathogen-reduction technologies, but alternatives are in development.

In 2008, the Advisory Committee on Blood and Tissue Safety and Availability (which advises the secretary of health and human services) determined that "accumulating evidence for the efficacy and safety of pathogen reduction warrants a commitment and concerted effort to add this technology as a broadly applicable safeguard." Yet despite data supporting the efficacy of such technologies in reducing or eliminating transfusion-transmitted pathogens, no mandate exists for their use.

The FDA has committed to releasing draft guidance in 2015 allowing men who have sex with men to donate blood after a 1-year period of sexual abstinence. Some observers have expressed concern that this change may increase the risk of transfusion-transmitted infections, despite current testing protocols. Use of pathogen-reduction technologies may mitigate these concerns by introducing an additional layer of safety. Similarly, bacterial contamination and associated septic transfusion reactions — a serious threat to platelet recipients since platelets are stored at room temperature, which favors bacterial growth — could most likely be eliminated through the use of licensed pathogen-reduction technologies.

Currently, blood-collecting facilities voluntarily test platelet units for contamination using FDA-approved culture techniques. Nevertheless, clinical sepsis is reported after 1 in every 100,000 platelet transfusions, and 1 in every 3000 units harbors clinically relevant bacterial concentrations. To address this substantial risk, the FDA recently released draft guidance requiring prerelease platelet screening for bacterial contamination and encouraging enhanced bacterial testing by hospitals (i.e., on the day of platelet transfusion), owing to the high false-negative rate during early screening (www.fda.gov/BiologicsBloodVaccines/GuidanceComplianceRegulatoryInformation/Guidances/default.htm). Because of cost and logistic considerations,

such recommendations will probably not be adopted unless they are mandated.

Current reactive screening for pathogens is limited by its failure to detect low levels of known transfusion-transmitted agents (e.g., human immunodeficiency virus [HIV] and hepatitis B and C viruses) soon after infection, during the so-called window period. Transfusion-transmitted HIV and hepatitis occur at rates of approximately 1 per 1 million units of transfused blood components. The use of pathogen-reduction technology would inactivate these viruses in blood components with low viral loads, closing the window period. A review of cases from countries with safeguards similar to those in the United States showed that 6 of 15 HIV transmissions and 12 of 19 hepatitis B virus transmissions in which the component type was documented were linked to platelets or plasma collected during the window period. Current estimates suggest that these reported cases represent a small fraction of actual transmissions.

In the past 15 years, blood centers have implemented numerous screening tests in response to transfusion threats. However, pathogens continue to emerge, and each incident calls transfusion safety into question. Potential threats include, but are not limited to, Ebola, dengue, chikungunya, hepatitis E, pandemic influenza, and SARS (severe acute respiratory syndrome) viruses. During the recent epidemics of dengue and chikungunya in the Caribbean, approximately 1 in every 500 blood donations was shown to contain viral RNA. Proactive pathogen reduction for platelets and plasma may defuse many emerging threats; with appropriate investment, we should someday be able to do the same for red cells. Continued use of a reactive approach to addressing each threat is not viable. With the use of pathogen-reduction technologies for platelets, plasma, and red cells, new screening tests would be needed only for pathogens that lack susceptibility to these techniques or that are present at concentrations exceeding the capacity of these techniques.

Critics believe that a policy mandating this process would increase the already high costs of donation screening. But with the adoption of pathogen-reduction techniques, certain screening tests, along with their costs, could be eliminated. Current users of this technology consider the resulting products to be safe with respect to bacteria, cytomegalovirus, and graft-versus-host disease, eliminating the need for bacterial detection, cytomegalovirus screening, and irradiation. Critics also raise concerns about the safety of adding chemicals to blood products, but few published reports have linked pathogen reduction to adverse events, and to date, no such finding has been reproducible.

Implementing new or improved technologies for bloodborne pathogens is difficult in an increasingly cost-constrained health care environment. Under the current

approach, recognition of a transfusion-transmitted pathogen triggers the diagnostics industry to invest in assay development, validation, and licensure. Blood centers that implement a new assay will typically not be able to recover the cost from hospitals until, often years later, the FDA decides to require testing by all centers.

Blood management and utilization programs ensuring that blood is used only when needed and in the smallest quantity possible have become widespread, but their adoption is a double-edged sword. Blood centers are facing a 20% decline in blood use, which translates into decreased cost recovery. Consequently, centers are downsizing infrastructure, reducing staff, closing facilities, and merging to remain fiscally sound. Individual centers are unable to absorb the additional costs of implementing new blood-safety interventions unless they are reimbursed by hospitals. Hospitals are not directly reimbursed for blood products and will purchase blood from the lowest-cost provider. All these factors inhibit the pursuit of safety innovations. Hospitals seeking to ensure their own fiscal solvency often view unmandated safety innovations as research they are not obligated to subsidize, an attitude that results in variable adoption of safety innovations and inconsistent safety standards.

Accrediting associations are finding it increasingly difficult to change practice by setting new standards that increase cost, without changes in reimbursement. Diagnostics manufacturers no longer view developing tests for voluntarily donated blood as commercially viable, since uncertainty regarding the scope of testing and testing mandates creates untenable investment risks. However, the ongoing safety of the U.S. blood supply relies on industry innovation, including the development of conventional screening assays, highly multiplexed testing platforms (e.g., next-generation sequencing and microarrays), and pathogen-reduction technologies.

The historical process of reactive, pathogen-specific test development is not sufficient to protect patients. The time has come for proactive pathogen reduction. Only the federal government can drive adoption by mandating universal implementation of available technologies. This mandate should be supported by a reimbursement process that recognizes the benefits of proactive strategies and offsets the costs. In addition, we believe that pathogen reduction for red-cell components should become a national research priority. We now have the means to protect patients from existing and emerging bloodborne threats — all we need is the will.

Tomorrow's Advance Man

Tad Friend, The New Yorker May 18, 2015 Issue

On a bright October morning, Suhail Doshi drove to Silicon Valley in his parents' Honda Civic, carrying a laptop with a twelve-slide presentation that was surely worth at least fifty million dollars. Doshi, the twenty-six-year-old C.E.O. of a data-analytics startup called Mixpanel, had come from San Francisco to Sand Hill Road in Menlo Park, where many of the world's most prestigious venture-capital firms cluster, to pitch Andreessen Horowitz, the road's newest and most unusual firm. Inside the offices, he stood at the head of a massive beechwood conference table to address the firm's deal team and its seven general partners—the men who venture the money, take a seat on the board, and fire the entrepreneur if things go wrong.

Marc Andreessen, the firm's co-founder, fixed his gaze on Doshi as he disinfected his germless hands with a sanitizing wipe. Andreessen is forty-three years old and six feet five inches tall, with a cranium so large, bald, and oblong that you can't help but think of words like "jumbo" and "Grade A." Two decades ago, he was the animating spirit of Netscape, the Web browser that launched the Internet boom. In many respects, he is the quintessential Silicon Valley venture capitalist: an imposing, fortyish, long-celebrated white man. (Forbes's Midas List of the top hundred V.C.s includes just five women.) But, whereas most V.C.s maintain a casual-Friday vibe, Andreessen seethes with beliefs. He's an evangelist for the church of technology, afire to reorder life as we know it. He believes that tech products will soon erase such primitive behaviors as paying cash (Bitcoin), eating cooked food (Soylent), and enduring a world unimproved by virtual reality (Oculus VR). He believes that Silicon Valley is mission control for mankind, which is therefore on a steep trajectory toward perfection. And when he so argues, fire-hosing you with syllogisms and data points and pre-refuting every potential rebuttal, he's very persuasive.

Doshi, lean and quizzical in a maroon T-shirt and jeans, began his pitch by declaring, "Most of the world will make decisions by either guessing or using their gut. They will be either lucky or wrong." Far better to apply Mixpanel's analytics, which enable mobile-based companies to know exactly who their customers are and how they use their apps. Doshi rapidly escalated to rhetoric—"We want to do data science for every single market in the world"—that would sound bumptious anywhere but on Sand Hill Road, where the young guy in jeans is obligated to astound the middle-aged guys in cashmere V-necks. "Mediocre V.C.s want to see that your company has traction," Doshi told me. "The top V.C.s want you to show them you can invent the future."

If you have a crackerjack idea, one of your stops on Sand Hill Road will be Andreessen Horowitz, often referred to by its alphanumeric URL, a16z. (There are sixteen letters between the “a” in Andreessen and the “z” in Horowitz.) Since the firm was launched, six years ago, it has vaulted into the top echelon of venture concerns. Competing V.C.s, disturbed by its speed and its power and the lavish prices it paid for deals, gave it another nickname: AHo. Each year, three thousand startups approach a16z with a “warm intro” from someone the firm knows. A16z invests in fifteen. Of those, at least ten will fold, three or four will prosper, and one might soar to be worth more than a billion dollars—a “unicorn,” in the local parlance. With great

luck, once a decade that unicorn will become a Google or a Facebook and return the V.C.'s money a thousand times over: the storied 1,000x. There are eight hundred and three V.C. firms in the U.S., and last year they spent forty-eight billion dollars chasing that dream.

Doshi had run the gantlet before. In 2012, he tracked down Andreessen and his equally if less splendidly bald co-founder, Ben Horowitz, at a Ritz-Carlton near Tucson. Then he pitched them in the lobby (having made sure that his parents' Honda, which contained his father, was well out of sight). Doshi mentioned that he'd become so dissatisfied with the incumbent database software that he'd built his own. Andreessen later told me that this "was like a cub reporter saying, 'I need to write the Great American Novel before I can really file this story.' " A16z gave Doshi ten million dollars, and he gave it twenty-five per cent of his company.

Now he was back for more. He zipped through his slides: hundred-per-cent growth rate; head count doubling every six to nine months; and he still had all the money he'd raised last time. As Andreessen drank an iced tea in two gulps and began to roam the room, Doshi called up a slide that showed his competitors—Localytics, Amplitude, Google Analytics—grouped into quadrants. Then he explained how he'd crush each quadrant. "I want to buy a machine-learning team, I want to buy cutting-edge server hardware," he said. Indicating his all-but-obliterated competitors, he added, "I want to buy stuff no one here can afford." He jammed his hands in his pockets: questions?

While entrepreneurs attack with historiography—"The great-man view of history is correct, and I am that great man!"—V.C.s defend with doubletalk. "You're definitely going to get funded!" means "But not by us." "Who else is in?" means "Besides not us." And "I'm not sure I would ever use your product myself" means "So long!" But the best V.C.s test the entrepreneur's mettle as well as their own assumptions. Andreessen gripped the back of his chair. "So one way to describe what you're doing is a network effect," he said. "More data gives you more customers, which allows you to build more services, which gives you more data, which allows you to get more customers, and you just turn the crank." Doshi thought this over and said, "Sure!" Andreessen grinned: he's a systems thinker, and he'd grasped how Mixpanel fit into the system. After the pitch, he told me that Mixpanel is "a picks-and-shovels business right in the middle of the gold rush."

When a startup is just an idea and a few employees, it looks for seed-round funding. When it has a product that early adopters like—or when it's run through its seed-round money—it tries to raise an A round. Once the product catches on, it's time for a B round, and on the rounds go. Most V.C.s contemplating an investment in one of

these early rounds consider the same factors. “The bottom seventy per cent of V.C.s just go down a checklist,” Jordan Cooper, a New York entrepreneur and V.C., said. “Monthly recurring revenue? Founder with experience? Good sales pipeline? X per cent of month-over-month growth?” V.C.s also pattern-match. If the kids are into Snapchat, fund things like it: Yik Yak, Streetchat, ooVoo. Or, at a slightly deeper level, if two dropouts from Stanford’s computer-science Ph.D. program created Google, fund more Stanford C.S.P. dropouts, because they blend superior capacity with monetizable dissatisfaction.

Venture capitalists with a knack for the 1,000x know that true innovations don’t follow a pattern. The future is always stranger than we expect: mobile phones and the Internet, not flying cars. Doug Leone, one of the leaders of Sequoia Capital, by consensus Silicon Valley’s top firm, said, “The biggest outcomes come when you break your previous mental model. The black-swan events of the past forty years—the PC, the router, the Internet, the iPhone—nobody had theses around those. So what’s useful to us is having Dumbo ears.”* A great V.C. keeps his ears pricked for a disturbing story with the elements of a fairy tale. This tale begins in another age (which happens to be the future), and features a lowborn hero who knows a secret from his hardscrabble experience. The hero encounters royalty (the V.C.s) who test him, and he harnesses magic (technology) to prevail. The tale ends in heaping treasure chests for all, borne home on the unicorn’s back.

At pitch meetings, Andreessen is relatively measured: he reserves his passion for the deal review afterward, when the firm decides whether to invest. That’s where he asks questions that oblige his partners to envision a new world. For the ride-sharing service Lyft: “Don’t think about how big the taxi market is. What if people no longer owned cars?” For OfferUp: “What if all this selling online—eBay and Craigslist—goes to mobile? How big could it be?” Ben Horowitz, who sits next to his co-founder at the head of the table, is an astute manager who quotes the rap lyrics of his friends Nas and Kanye West to inspire fearless thinking—but he doesn’t try to manage Andreessen. “If you say to Marc, ‘Don’t bite somebody’s fucking head off!,’ that would be wrong,” Horowitz said. “Because a lot of his value, when you’re making giant decisions for huge amounts of money, is saying, ‘Why aren’t you fucking considering this and this and this?’ ”

A16z was designed to be a full-throated argument about the future, a design predicated on its founders’ comfort with conflict. In 1996, when Horowitz was a Netscape product manager, he wrote a note to Andreessen, accusing him of prematurely revealing the company’s new strategy to a reporter. Andreessen wrote back to say that it would be Horowitz’s fault if the company failed: “Next time do the fucking interview yourself. Fuck you.” Ordinarily, relationship over. “When he feels disrespected,

Marc can cut you out of his life like a cancer," one of Andreessen's close friends said. "But Ben and Marc fight like cats and dogs, then forget about it." Two years later, when Netscape was floundering and forty per cent of its employees left, Horowitz announced that he was staying no matter what. Andreessen had never trusted anyone before, but he began to consider it. Their teamwork at a16z is complementary: Horowitz is the people-person C.E.O., and Andreessen is the farsighted theorist, the chairman. Yet Horowitz noted that "Marc is much more sensitive than I am, actually. He'll get upset about my body language—'God damn it, Ben, you look like you're going to throw up when I'm talking about this!' "

Although Andreessen has been a board member of Facebook, Hewlett-Packard, and eBay, he doesn't take many board seats in a16z's portfolio companies, preferring to train his eyes on the horizon. Andreessen is tomorrow's advance man, routinely laying out "what will happen in the next ten, twenty, thirty years," as if he were glancing at his Google calendar. He views his acuity as a matter of careful observation and extrapolation, and often invokes William Gibson's observation "The future is already here—it's just not very evenly distributed." Jet packs have been around for half a century, but you still can't buy them at Target. To smooth out such lumps in distribution, Andreessen disseminates his views via every available podcast and panel discussion and CNN interview slot: he's a media soothsayer, Andreessen the Magnificent. He also tweets a hundred and ten times a day, inundating his three hundred and ten thousand followers with aphorisms and statistics and tweetstorm jeremiads. Andreessen says that he loves Twitter because "reporters are obsessed with it. It's like a tube and I have loudspeakers installed in every reporting cubicle around the world." He believes that if you say it often enough and insistently enough it will come—a glorious revenge. He told me, "We have this theory of nerd nation, of forty or fifty million people all over the world who believe that other nerds have more in common with them than the people in their own country. So you get to choose what tribe or band or group you're a part of." The nation-states of Twitter will map the world.

Mixpanel was emblematic of Silicon Valley's outsized worship of unicorns. At the company's deal review, Peter Levine, who sits on Doshi's board, reported that the entrepreneur had e-mailed to say that he'd love for his company to be valued at a billion dollars—an assessment that would set the price for the portion of it that a16z might now buy. However, Doshi would sell the firm ten per cent of his company for eighty million, suggesting a valuation of eight hundred million dollars. Andreessen said, "The dogs are fucking jumping through the screen door to eat the dog food. And he hasn't done any marketing yet. And he's profitable!"

Horowitz exclaimed, "How old is he, twenty-four? God damn it, let's give him all our

money!" A16z provided Doshi all his B-round funding—sixty-five million dollars—for a further 7.5 per cent of the company, which was thus valued at eight hundred and sixty-five million dollars. Doshi was a little sorry that Mixpanel wasn't valued at a billion dollars, but he told me that he could wait: his business was growing so fast, and everyone was raising money so frequently in the current boom, that "in six or twelve months we'll be a unicorn."

Venture firms rarely do an entire follow-on round themselves, for fear of losing sight of a company's true market value; as Andreessen put it, "You can be thinking your shit smells like ice cream." None of the half-dozen other firms that Doshi pitched last fall valued his company as highly as a16z did. But Andreessen applied a maxim from his friend and intellectual sparring partner Peter Thiel, who co-founded PayPal and was an early investor in LinkedIn and Yelp. When a reputable venture firm leads two consecutive rounds of investment in a company, Andreessen told me, Thiel believes that that is "a screaming buy signal, and the bigger the markup on the last round the more undervalued the company is." Thiel's point, which takes a moment to digest, is that, when a company grows extremely rapidly, even its bullish V.C.s, having recently set a relatively low value on the previous round, will be slightly stuck in the past. The faster the growth, the farther behind they'll be. Andreessen grinned, appreciating the paradox: the more they paid for Mixpanel—according to Thiel, anyway—the better a deal they'd be getting.

Most businesses don't work like this. At least, not yet.

Silicon Valley, the fifteen-hundred- square-mile shelf an hour south of San Francisco, was called the Santa Clara Valley until the rise of the microprocessor, in the nineteen-seventies. It remains contested ground. Armies of startups attack every incumbent, with early employees—and sometimes even their lawyers and landlords—taking deferred compensation, in the hope that their options and warrants will pay off down the line. Yet workers' loyalty is not to a company or even to an idea but to the iterative promise of the region. "Uber is built on the efforts of thousands of people in the Valley," the investor Naval Ravikant said. "On the back of the iPhone and Android and G.P.S. and battery technology and online credit-card payments, all stacked on themselves."

V.C.s give the Valley its continuity—and its ammunition. They are the arms merchants who can turn your crazy idea and your expendable youth into a team of coders with Thunderbolt monitors. Apple and Microsoft got started with venture money; so did Starbucks, the Home Depot, Whole Foods Market, and JetBlue. V.C.s made their key introductions and stole from every page of Sun Tzu to help them penetrate markets. And yet V.C.s maintain a zone of embarrassed privacy around their activ-

ities. They tell strangers they're investors, or work in technology, because, in a Valley that valorizes the entrepreneur, they don't want to be seen as just the money. "I say I'm in the software industry," one of the Valley's best-known V.C.s told me. "I'm ashamed of the truth."

At a hundred and eleven dollars a square foot, Sand Hill Road is America's most expensive office-rental market—an oak-and-eucalyptus-lined prospect stippled with bland, two-story ski chalets constrained by an ethos of nonconspicuous consumption (except for the Teslas in the parking lot). It's a community of paranoid optimists. The top firms coöperate and compete by turns, suspicious of any company whose previous round wasn't led by another top-five firm even as they're jealous of that firm for leading it. They call this Schadenfreude-riddled relationship "co-opitition." Firms trumpet their boldness, yet they often follow one another, lemming-like, pursuing the latest innovation—pen-based computers, biotech, interactive television, superconductors, clean tech—off a cliff.

Venture capital became a profession here when an investor named Arthur Rock bankrolled Intel, in 1968. Intel's co-founder Gordon Moore coined the phrase "vulture capital," because V.C.s could pick you clean. Semiretired millionaires who routinely arrived late for pitch meetings, they'd take half your company and replace you with a C.E.O. of their choosing—if you were lucky. But V.C.s can also anoint you. The imprimatur of a top firm's investment is so powerful that entrepreneurs routinely accept a twenty-five per cent lower valuation to get it. Patrick Collison, a co-founder of the online-payment company Stripe, says that landing Sequoia, Peter Thiel, and a16z as seed investors "was a signal that was not lost on the banks we wanted to work with." Laughing, he noted that the valuation in the next round of funding—"for a pre-launch company from very untested entrepreneurs who had very few customers"—was a hundred million dollars. Stewart Butterfield, a co-founder of the office-messaging app Slack, told me, "It's hard to overestimate how much the perception of the quality of the V.C. firm you're with matters—the signal it sends to other V.C.s, to potential employees, to customers, to the tech press. It's like where you went to college."

A venture firm musters its ammunition—say, a fund of a hundred and fifty million dollars—by recruiting investors such as university endowments and pension funds to become "limited partners," or L.P.s, in the fund. The firm invests the money for three or four years, then harvests the returns for the remainder of the fund's ten-year term. In theory, V.C.s, like entrepreneurs, are motivated by delayed gratification. The standard fee is "two and twenty": two per cent of the fund each year, and twenty per cent of the ultimate profits. (The top firms, including a16z, charge thirty per cent.) L.P.s expect returns equal to at least those they'd get in the stock market,

plus an additional five per cent for the illiquidity of the investment. For top firms, the dream is 5x to 10x.

At the moment, venture funding accounts for less than 0.3 per cent of the U.S.'s G.D.P. "Venture is often called a rounding error in the economy," Herbert Allen III, the head of the investment bank Allen & Company, said. "But the bang for the buck is huge. And venture is a major source of the optimism that underlies the American myth." Venture speeds the cycle of American impatience: what exists is bad and what replaces it is good—until the new thing itself must be supplanted.

Corporate culture, civic responsibility, becoming a pillar of society—these are not venture's concerns. Andy Weissman, a partner at New York's Union Square Ventures, noted that venture in the Valley is a perfect embodiment of the capitalist dynamic that the economist Joseph Schumpeter called "creative destruction." Weissman said, "Silicon Valley V.C.s are all techno-optimists. They have the arrogant belief that you can take a geography and remove all obstructions and have nothing but a free flow of capital and ideas, and that it's good, it's very good, to creatively destroy everything that has gone before." Some Silicon Valley V.C.s believe that these values would have greater sway if their community left America behind: Andreessen's nerd nation with a charter and a geographic locale. Peter Thiel favors "seasteading," establishing floating cities in the middle of the ocean. Balaji Srinivasan, until recently a general partner at a16z and now the chairman of one of its Bitcoin companies, has called for the "ultimate exit." Arguing that the United States is as fossilized as Microsoft, and that the Valley has become stronger than Boston, New York, Los Angeles, and Washington, D.C., combined, Srinivasan believes that its denizens should "build an opt-in society, ultimately outside the U.S., run by technology."

The game in Silicon Valley, while it remains part of California, is not ferocious intelligence or a contrarian investment thesis: everyone has that. It's not even wealth: anyone can become a billionaire just by rooming with Mark Zuckerberg. It's prescience. And then it's removing every obstacle to the ferocious clarity of your vision: incumbents, regulations, folkways, people. Can you not just see the future but summon it?

Marc Andreessen mentions Thomas Edison often, his family never. When he was growing up, outside the no-stoplight town of New Lisbon, Wisconsin, his father, Lowell, was a sales manager for a seed company called Pioneer Hi-Bred International, and his mother, Pat, worked in customer service at Lands' End—but I didn't get that information from him. A friend who knows Andreessen well told me, "We've never had a conversation about his parents or his brother—all he said was 'They didn't like me, and I didn't like them all that much, either.' "

The few details Andreessen let slip to me suggested a climate of antiquity, superstition, frustration, and penury. "The natural state of human beings is to be subsistence farmers, and that was my expectation," he said, adding that his world was "Scandinavian, hard-core, very self-denying people who go through life never expecting to be happy." The family telephone was a party line, and the bathroom at his relatives' farm was an outhouse. Everyone believed in dowsing and the weather reports in the Farmers' Almanac. One winter, with money tight, his father decided to stop paying for gas heat, "and we spent a great deal of time chopping fucking wood." The local movie theatre, one town over, was an unheated room that doubled as a fertilizer-storage depot; Andreessen wore a puffy Pioneer Hi-Bred coat to watch "Star Wars" while sitting on the makings of a huge bomb. He had to drive an hour to find a Waldenbooks, in La Crosse; it was all cookbooks and cat calendars. So he later saw Amazon as a heroic disseminator of knowledge and progress. "Screw the independent bookstores," he told me. "There weren't any near where I grew up. There were only ones in college towns. The rest of us could go pound sand."

Andreessen's vision of the future, and of his escape route, came from television. He told me, "KITT, the car in 'Knight Rider,' was a computer that could analyze a poison-gas attack. The car was magic—but now you can actually do all those things. A new car isn't KITT, but it does have all the maps and all the music in the world, and it talks to you. Even the transporter beam in 'Star Trek' basically makes sense if you understand quantum entanglement. People are composed of quantum elements, so there is a path!"

Something of the transporter beam clings to Andreessen, a sense that he just rematerialized from a city on the edge of forever. He's not great at the basics of daily life: directions confound him, because roadways aren't logical, and he's so absent-minded about sunglasses that he keeps a "reload station" with nine pairs on his hall table. Perhaps Edison haunts his conversation because Andreessen is a fellow-tinkerer, except that his gadgets are systems and platforms, and his workshop is his own mind. He regularly reprograms his appearance and deportment—his user interface—to suit his present role, and friends refer to chapters in his life as versions of an operating system: "Marc 1.0," "Marc 2.0," and so on. A charismatic introvert, Andreessen draws people in but doesn't really want them around. Though he has a crisp sense of humor, it's rarely deployed at his own expense. He hates being complimented, looked at, or embraced, and has toyed with the idea of wearing a T-shirt that says "No hugging, no touching." He doesn't grasp the protocols of social chitchat, and prefers getting a memo to which he can e-mail a response, typing at a hundred and forty words a minute. He didn't attend Netscape's twentieth-anniversary celebration, because it combined two things from which he recoils: parties and reminisc-

ing.

Yet he's also energetic and decisive, which makes him a valued counsellor. In 2006, Yahoo! offered to buy Facebook for a billion dollars, and Accel Partners, Facebook's lead investor, urged Mark Zuckerberg to accept. Andreessen said, "Every single person involved in Facebook wanted Mark to take the Yahoo! offer. The psychological pressure they put on this twenty-two-year-old was intense. Mark and I really bonded in that period, because I told him, 'Don't sell, don't sell, don't sell!' " Zuckerberg told me, "Marc has this really deep belief that when companies are executing well on their vision they can have a much bigger effect on the world than people think, not just as a business but as a steward of humanity—if they have the time to execute." He didn't sell; Facebook is now worth two hundred and eighteen billion dollars.

Andreessen's range of reference extends from Ibn Khaldun to "South Park," yet he approaches new topics as if starved, eating through men's fashion or whiskey-making or congressional politics until it has yielded every micronutrient. In a tweetstorm about the question of net neutrality, he observed that anyone who took a position should be versed in the "history, technology, and economics of backbones, interconnection agreement, peering, CDNs, caching, colocation, current and future telco and cable business models including capex and opex models, rate caps, cost of capital, return on investment," as well as a dozen other equally abstruse matters. He coyly noted that no one, himself included, understood them all—then stated his position. Andreessen's learning fuses the idiosyncrasy of the autodidact with the thoroughness of what programmers call depth-first search. "I could never tolerate not knowing why," he said. "You have to work your way back to figure out the politics, the motivations. I always stop when I get to evolutionary psychology, and why we have tribes—oh, O.K., we're primates cursed with emotions and the ability to do logical thinking." He keeps rediscovering that we're australopithecines, and keeps hoping to transform us into Homo habilis: man the tool user, able man.

To this end, he addresses any topic, such as Google's purchase of the thermostat maker Nest, by launching a dialectics—"1) Either Nest is the most amazing company ever, or 2) Larry Page acqui-hired Tony Fadell for $3.2 billion and got a thermostat business on the side"—whose synthesis is often that the thesis and the antithesis were simplistic ("Or, maybe Google has a larger plan for automating the home") or irrelevant ("Whatever, whatever, we don't own it, so who cares?"). Often, he discourses at such lucid length that his cheeks redden and he must pause for breath. If you seize the interval to demonstrate a basic grasp of his argument, he'll say "Exzact-ly," with a pleased smile, and upload another tranche. What saves him from pompous know-it-all-dom, most of the time, is this eagerness to communicate.

He turns to theory the way a drinker turns to the minibar. But Horowitz told me that every once in a while Andreessen will "get all Wisconsin on you, sticking up for his people. When we looked at an Internet pawnshop, people here said, 'It's immoral,' and Marc went bananas. He said, 'If you've got no fucking money, and you need to pawn your watch to pay for your kids to eat—you think that's morally fucking wrong because it offends your sensibilities, you rich motherfuckers?' He knew that guy who was pawning his watch because he'd missed the harvest, or whatever. Or we saw an Uber-for-private-jets thing, or some wine thing that came through, and he just got incensed: 'We didn't start the firm for rich people to buy hundred-dollar bottles of wine or to fly around on fucking private jets!' He reminds me of Kanye, that level of emotional intensity—his childhood was so intensely bad he just won't go there."

One afternoon, Alexis Ringwald, the C.E.O. of LearnUp, a job-training startup that has worked with Staples and Old Navy, stood in a16z's conference room, all poise and smile. "I like to launch movements to tackle huge problems," she said, launching into her presentation.

"Start at the beginning, where you grew up," Ben Horowitz said. A16z had made a small seed investment in Ringwald's company, but most of the general partners, who were about to tell her whether she was ready for an A round, didn't know much about her. Horowitz also routinely forces a founder to abandon her script and regroup. It's a stress test intended to elicit biography, resilience, and the real story.

Ringwald, who is thirty-one, blinked, then shifted smoothly to an engaging account of her early years, her work interviewing people on the unemployment line, and how she'd eventually realized that the country's biggest gulf is between those who have the basic skills to be employable—showing up on time, dressing neatly—and those who don't. "So it's a modern 'My Fair Lady' sort of thing?" Horowitz asked, ingenuously. Ringwald crisply noted that her process triples an applicant's chance of getting a job, and that eighty-two per cent of LearnUp's trainees outperform their fellow-workers. Horowitz and Andreessen nodded: she could handle the pressure. Afterward, Horowitz told me, "My big conclusion was she's a legit Pied Piper, with charisma and will and fury."

Pitch meetings are minefields. If a V.C. asks you, "When you get to a hundred engineers, are you worried about the company culture or excited?," the correct answer is "A hundred? I want a thousand!" Reid Hoffman, a V.C. at Greylock Partners who co-founded LinkedIn, told me, "I look to see if someone has a marine strategy, for taking the beach; an army strategy, for taking the country; and a police strategy, for

governing the country afterward."

A16z wants to learn if the founder has a secret—a novel insight, drawn from personal experience, about how the world could be better arranged. If that new arrangement is 10x better, consumers might be won over. Balaji Srinivasan contributed the concept of the "idea maze": you want the entrepreneur to have spent years thinking her idea into—and out of—every conceivable dead end. "Entrepreneurs want to raise money from us," Andreessen told me, "so the natural thing when we say 'What if you did this?' is to tell us what we want to hear. But we don't want to hear what we want to hear. It's a delight when they look at you with contempt—You idiot—and then walk you through the idea maze and explain why your idea won't work." Such tests help a16z determine whether the founder is a mercenary who wants to sell the company within four years, which will cap a16z's return at 5x, or a missionary, determined to change the world. "At the same time," Andreessen said, "we're not funding Mother Teresa. We're funding imperial, will-to-power people who want to crush their competition. Companies can only have a big impact on the world if they get big."

Ringwald, back into her planned remarks, promised bigness: "LearnUp will transform employment in America. We can unleash human potential and move the needle on the G.D.P." Andreessen said, "Question: This is a known problem. Why do companies not just do this themselves, once they see that it works?" Ringwald replied, "We'll keep on differentiating by moving fast and collecting more data on what companies need now."

Then a general partner named Chris Dixon asked, "Is it a marketplace or an enterprise company?" Marketplace companies sell to consumers; enterprise companies sell to other businesses. Clearly perplexed by the distinction, Ringwald said that she was signing up workers as well as companies. Everyone became a shade more remote.

Afterward, Andreessen told his colleagues, "She didn't really answer Chris's question. If it's marketplace, it's defensible; if it's enterprise, she can be undercut." If Ringwald's customers were the workers, who would keep using LearnUp as they moved from job to job, she could create a network effect. If her customers were actually the companies, they could start doing the training themselves—or another startup could. A16z views marketplace and enterprise companies very differently. The firm invests early with enterprise, but waits with consumer companies, because they tend to take off—suddenly, everyone wants to be on Instagram—or fail fast. It's a risk-averse way to embrace risk. In 2013, a16z passed on the A round of Oculus VR (waiting to see if it could resolve the nausea issue that has plagued virtual-reality

systems) and came in on the B, six months later. It got the same ten per cent of the company it could have had in the A—but it paid thirty million dollars instead of six million. The internal rationale for this expensive "de-risking" is "We paid up for certainty."

The partners began to discuss how LearnUp might be valued. Valuation, particularly in a company's early rounds, often derives less from spreadsheets than from market forces—what are other firms offering?—and the "What if"s of mental modelling. Does the company's traction, leadership team, and "total addressable market" call to mind a Pinterest, or does it feel more like a ShoeDazzle? One partner suggested that LearnUp was a "ten on thirty"—ten million dollars should buy a third of the company, which would then be valued at forty million. "It's more like ten on fifteen or twenty," Horowitz said, cutting the company's value in half. "Or six on twelve," Andreessen said, whittling it further. Soon after the meeting, Ringwald turned LearnUp into an enterprise company.

Most venture firms operate as a guild; each partner works with his own companies, and a small shared staff helps with business development and recruiting. A16z introduced a new model: the venture company. Its general partners make about three hundred thousand dollars a year, far less than the industry standard of at least a million dollars, and the savings pays for sixty-five specialists in executive talent, tech talent, market development, corporate development, and marketing. A16z maintains a network of twenty thousand contacts and brings two thousand established companies a year to its executive briefing center to meet its startups (which has produced a pipeline of deals worth three billion dollars). Andreessen told me, "We give our founders the networking superpower, hyper-accelerating someone into a fully functional C.E.O. in five years."

The firm's fourteen-person deal team also enables it to rapidly assess any new technology, making a16z a kind of Iron Man suit for Andreessen as he pursues his flights of fancy. Jim Breyer, who led Facebook's first venture round at Accel Partners, told me, "I spend most of my time trying to connect the dots for what the future will look like in five to seven years, but I don't believe I scale as well as Marc and Ben and their team. They've moved into next-gen agricultural products and wearables and drone software, where a lot of us don't have expertise or networks."

Andreessen and Horowitz launched the firm in 2009, when venture investment was frozen by the recession. Their strategy was shaped by their friend Andy Rachleff, a former V.C. He told them that he'd run the numbers and that fifteen technology companies a year reach a hundred million dollars in annual revenue—and they account for ninety-eight per cent of the market capitalization of companies that go

public. So a16z had to get those fifteen companies to pitch them. “Deal flow is everything, ” Andreessen told me. “If you’re in a second-tier firm, you never get a chance at that great company.” A leading investment banker who has taken numerous software companies public told me, “I put ninety per cent of my effort into seeking out deals from the top eight venture firms, ten per cent into the next twelve, and zero per cent into all the rest.”

The dirty secret of the trade is that the bottom three-quarters of venture firms didn’t beat the Nasdaq for the past five years. In a stinging 2012 report, the L.P. Diane Mulcahy calculated, “Since 1997, less cash has been returned to V.C. investors than they have invested.” The truth is that most V.C.s subsist entirely on fees, which they compound by raising a new fund every three years. Returns are kept hidden by nondisclosure agreements, and so V.C.s routinely overstate them, both to encourage investment and to attract entrepreneurs. “You can’t find a venture fund anywhere that’s not in the top quartile,” one L.P. said sardonically. V.C.s also logo shop, buying into late rounds of hot companies at high prices so they can list them on their portfolio page.

When a16z began, it didn’t have even an ersatz track record to promote. So Andreessen and Horowitz consulted on tactics with their friend Michael Ovitz, who co-founded the Hollywood talent agency Creative Artists Agency, in 1974. Ovitz told me that he’d advised them to distinguish themselves by treating the entrepreneur as a client: “Take the long view of your platform, rather than a transactional one. Call everyone a partner, offer services the others don’t, and help people who aren’t your clients. Disrupt to differentiate by becoming a dream-execution machine.”

Believing that founders make the best C.E.O.s—look at Intel, Apple, Oracle, Google, Facebook—Andreessen and Horowitz recruited only general partners who’d been founders or run companies. Then they began constructing the illusion of authority, taking offices on Sand Hill Road and filling them with paintings by Robert Rauschenberg and Sol LeWitt—another page from the book of Ovitz, who commissioned a Roy Lichtenstein painting for C.A.A.’s lobby that was so large the firm had to leave it behind when it moved. They were studiously punctual (partners are fined ten dollars for each minute they’re late to a pitch), used glassware rather than plastic, and said no quickly and explained why (unless the reason was doubts about the entrepreneur) in a handwritten note. And, while most V.C.s were publicity averse—Sequoia’s slogan was “The entrepreneurs behind the entrepreneurs”—a16z banged the drum to draw startups. The tech publicist Margit Wennmachers built an eight-person marketing department and helped to orchestrate stories in Forbes and Fortune.

Andreessen and Horowitz believed that it would take them years to get great deal

flow. So instead of fighting for A-round financings—the most competitive round, because it's when you can buy the largest chunk of an up-and-coming company—they planned to make seed investments in eighty startups. They wouldn't take the customary board seats (otherwise, they'd each be sitting on forty boards), but they'd help all eighty companies and then lead the A round for the twelve best.

The strategy had flaws. Entrepreneurs want V.C.s on their boards, and so do L.P.s: that's how you really learn a company. The firm would be sending a huge negative signal about companies it didn't reinvest in—hardly an entrepreneur-friendly stance. Furthermore, by making so many investments, a16z would create significant opportunity costs. In its first year, it put two hundred and fifty thousand dollars into a company called Burbn, which soon pivoted and became Instagram—but a16z couldn't increase its share, because it had also taken a position in a short-lived photo app called PicPlz. Though the firm made 312x when Facebook bought Instagram, the huge multiple amounted to only seventy-eight million dollars. Elizabeth Obershaw, a managing director at Horsley Bridge, a prominent L.P. that invested in a16z after some debate, told me, "Our list of cons was that we didn't think their original model would work at all. The pros were Marc and Ben—we decided they were learners and adapters and would realize the model wasn't workable fast enough to fix it—and an industry that was ripe for reinvention."

They learned fast. After a16z raised a three-hundred-million-dollar fund and opened shop, in July, 2009, it did a lot of seed rounds, but it also spent fifty million dollars to buy three per cent of Skype. Two years later, Microsoft bought Skype, and the investment returned 4x. Andreessen believed that everyone had underestimated the size of the Internet market, so in 2010, after raising a much bigger second fund, the firm spent a hundred and thirty million dollars to acquire shares of Facebook and Twitter at unprecedented valuations. Other V.C.s sniped that a16z was trying to buy its way in: Skype was an established company, not a startup, and the Facebook and Twitter deals were mere logo shopping. But, as Ron Conway, Silicon Valley's leading angel investor, noted, "In twenty-four months, Andreessen Horowitz was the talk of the town." The firm won a hundred-million-dollar A round for the coding company GitHub, which Conway called "the most hotly contested deal in five years." Chris Wanstrath, GitHub's co-founder and C.E.O., said that a16z's services were a major attraction: "It's like a buffet—they offered a bunch of great dishes, and we wanted to sample them all."

After six years, Andreessen believes, a16z is meeting—and winning—enough new clients to place it "comfortably in the top three" V.C. firms. (This is not far off from the consensus in the Valley.) Its first fund has already returned 2x, and contains such powerhouses as Slack and the identity-management company Okta. The fund's

internal rate of return, a calculation of annualized profit, is fifty per cent, which places it very high among funds raised in 2009. (Sequoia's rate for its corresponding fund is sixty-nine per cent.) The firm's second fund includes Pinterest and Airbnb, and its third fund includes Zenefits, GitHub, and Mixpanel; both funds, on paper, are well into the black. A respected L.P. of the firm told me, "They're one of our top performers." Yet Andreessen cautioned, "We still have a lot to prove on returns. I wouldn't be comfortable saying we're No. 1 until ten years have passed, maybe fifteen. Until then, it's Schrödinger's cat, and I've got really good arguments on why the cats are both alive and dead."

At Andreessen's wedding, in 2006, Ben Horowitz said in his toast that the man he'd long known was "grouchy Marc," because he'd "gone through his whole life without anyone understanding him, being all by himself." No one had understood him in his farm town, no one had understood him in Silicon Valley—"Hell, I do not understand him." But now, at last, he was "happy Marc," because he'd found "someone who totally gets him": the bride, a lecturer in philanthropy at Stanford's business school named Laura Arrillaga-Andreessen.

In December, Andreessen invited me to their house in Atherton, five minutes from a16z's office, to watch television. He and Laura live in a modern, art-filled, nine-thousand-square-foot villa built in a style that she calls "Northern California pastiche." The ceilings are scaled to Andreessen's Brobdingnagian proportions, and everything is majestic, minimal, and new. The toilet in the powder room is so visionary, and the surrounding dimmer lights so flattering, that I had to study it for some time to figure out how it flushed.

Arrillaga-Andreessen brought the couple's dinners into the living room and placed them on matching Costco TV tables. The omelettes and Thai salads that their chef had prepared earlier had been freshly reheated (they have three microwaves, so their food will always be ready at the same time). Andreessen stroked her arm and beamed: "Hello, gorgeous!"

"Hello, my darling!" she replied. Then she gave me a dramatic hug, as we hadn't seen each other since the previous day. Arrillaga-Andreessen is a tall, ethereal-seeming, yet effusive woman. When the couple met, in 2005, at a New Year's Eve dinner thrown by the leading investor in eHarmony, they talked for six and a half hours. She told me that Andreessen satisfied most of the criteria on her checklist: he was a genius, he was a coder, he was funny, and he was bald. ("I find it incredibly sexy to see the encasement of a cerebrum," she explained.) For his part, Andreessen felt that "she was spectacular! My biggest concern was that she wanted to live a jet-set life." In one of the seventeen e-mails he sent her the next day, he asked, "What's your

ideal evening?" She responded, "Stay home, do e-mail, make an omelette, watch TV, take a bath, go to bed." Before their second date, he delivered what she calls "a twenty-five-minute monologue on why we should go steady, with a full intellectual decision tree in anticipation of my own decision tree." They were married nine months later. In her and her father, John, a billionaire Silicon Valley developer, Andreessen seems to have found a replacement family. Laura showed me a photograph of the two men side by side, both bald, self-made, and magisterial: "Quite two peas in a pod."

After some TV time together, the couple reads in bed, so that, she says, "I can fall asleep holding my beloved." (She invariably refers to her husband as "my beloved," rather than "Marc.") "I ask him questions about things I got curious about during the day, so every night I'm going to sleep with a human Wikipedia that can go deeper and deeper and deeper, link upon link. In the past week, we talked about all the hardware components of a mobile phone, how binary code works, what might happen with drone regulation, and whether Putin is using Ukraine as a distraction from the financial crisis in Russia." Once she's dozed off, Andreessen returns to work in his home office, where, like a recharging cell phone, he gains energy through the night.

He pushed a button to unroll the wall screen, then called up Apple TV. We were going to watch the final two episodes of the first season of the AMC drama "Halt and Catch Fire," about a fictional company called Cardiff, which enters the personal-computer wars of the early eighties. The show's resonance for Andreessen was plain. In 1983, he said, "I was twelve, and I didn't know anything about startups or venture capital, but I knew all the products." He used the school library's Radio Shack TRS-80 to build a calculator for math homework. In 1992, as an undergraduate at the University of Illinois at Urbana-Champaign, he neglected his job—writing Unix code for $6.85 an hour—to team with another programmer to create Mosaic, the first graphical browser for the Web. After graduating, he moved to Silicon Valley, where he and a volatile serial entrepreneur named Jim Clark launched Netscape, to make the Internet available not just to scientists but to everyone. John Doerr, the V.C. who funded their A round, said that the genius of their browser was that "it was like putting photos on the menu at Howard Johnson. You didn't need to know the language; you could just point." The story underlying that story, Arrillaga-Andreessen told me—the secret—was that "Netscape was based on my beloved's own inability, as a child, to access knowledge in a small town."

Netscape Navigator, released in 1994, quickly claimed more than ninety per cent of the browser market, and Andreessen predicted that the Web would make operating systems such as Microsoft's Windows "irrelevant." When the company went public,

in 1995, its stock rocketed from twenty-eight dollars a share to seventy-five dollars, and Andreessen was soon on the cover of Time, barefoot on a throne. But Marc 1.0 was very much in beta. Having given up coding, his first love, to manage coders, he scarfed Pepperidge Farm Nantuckets and Honeycomb cereal straight from the box, skipped meetings, and blazed up without warning. "You'd see him vibrating, and it would inspire a combination of excitement and terror," Jason Rosenthal, a manager whom Andreessen actually liked, recalled. A favorite Andreessen response to underlings' confusion was "There are no stupid questions, only stupid people." Jim Barksdale, the company's C.E.O., said, "I'd tell Marc after meetings, 'You don't have to tell a dumb sumbitch he's a dumb sumbitch.' " Andreessen told me, "I needed Netscape to work, it had to work—it was my one-way door—so I was absolutely intolerant of anything that got in the way"—meaning, he clarified, "people." He could never relax: "I am very paranoid. And the down cycle hurt a lot more than the up cycle felt good."

The down cycle began when Microsoft bundled its own browser with its operating system, making it the nation's browser of convenience, if not of choice. Netscape shifted from marketplace to enterprise, and began selling browser and server software, but it was fortunate to get bought by AOL, in 1999, for ten billion dollars. Peter Currie, the company's C.F.O., said, "We made a difference, we invented cookies and pioneered downloading software from the Internet, yet Netscape is an asterisk in business history. Maybe the best way to think about it is as a classic tech story: a company creates, invents, succeeds—and gets bypassed."

In the first "Halt and Catch Fire" episode, Cardiff's entrepreneurs go to Comdex, the big trade show, and discover that another company has stolen their idea and beaten them to market. In response, Gordon, the hardware engineer, removes the interactive operating system from their Cardiff machine—a system designed by Cameron, a punk female software prodigy—and slots in Microsoft's dos, which makes the machine I.B.M.-compatible, viable, and dull. It was an excruciating capitulation, but Andreessen nodded evenly: "This was Microsoft's moment, and Gordon is right—they need to live to fight another day. But . . ." He pointed at the screen, where Apple's Macintosh was making its début at the trade show. "Hello, I'm Macintosh," the machine said. Andreessen laughed and continued, "They were doomed from the start, because Apple in Cupertino"—in Silicon Valley—"had spent three years building that. I've been totally determined to be on the other side of that dynamic by being here, because success in software follows a power-law distribution. It's not Coke and Pepsi and a bunch of others; it's winner take all. Second prize is a set of steak knives, and third prize is you're fired."

In the season finale, Cameron launches her own startup. As Andreessen watched her manage her coders, he said, softly, "The best scenes with Cameron were when

she was alone in the basement, coding." I said I felt that she was the least satisfactory character: underwritten, inconsistent, lacking in plausible motivation. He smiled and replied, "Because she's the future."

In "Why Software Is Eating the World," a widely invoked 2011 op-ed in the Wall Street Journal, Andreessen put the most optimistic spin on Silicon Valley's tendencies. The article proclaimed that tech companies are consuming vast swaths of the economy, from books and movies to financial services to agriculture to national defense—which Andreessen saw as the healthful scavenging of a carrion way of life. On Twitter, he pursued the theme: "Posit a world in which all material needs are provided free, by robots and material synthesizers. . . . Imagine six, or 10, billion people doing nothing but arts and sciences, culture and exploring and learning. What a world that would be," particularly as "technological progress is precisely what makes a strong, rigorous social safety net affordable."

Andreessen's telepathic method—extrapolating the future from current trends—may be the best available, but it has had doubtful results. Of the eighteen firms that V.C.s valued at more than a billion dollars in the heady days of 1999-2000, eleven have gone out of business or have been liquidated in fire sales, including @Home, eToys, and Webvan. A16z bought into Zulily, an online marketer, at a valuation of a billion dollars; it soared to a market capitalization of five billion dollars, and has since slumped to $1.3 billion. Another billion-dollar a16z company, the bargain-shopping site Fab, recently sold for about thirty million dollars. On the other hand, the firm wrote off the gaming company Slack to zero—and then it became an office-messaging app that's now valued at $2.8 billion.

The random, contingent way that the future comes to pass is a source of endless frustration in the Valley. Sam Altman, the president of the startup incubator Y Combinator, notes that his early investment in Stripe is now worth, on paper, more than 2,000x. "So ninety-seven per cent of my returns from 2010 and 2011 are concentrated in one investment, which I could easily have missed," he said. "I only let myself think about this sort of thing on vacation, because if I acknowledged that I was wasting more than ninety per cent of my time—which is true, from an economic perspective—I couldn't get through my days."

The key to investing, Andreessen contends, is to be aggressive and to fight your instinct to pattern-match. "Breakthrough ideas look crazy, nuts," he said, adding, "It's hard to think this way—I see it in other people's body language, and I can feel it in my own, where I sometimes feel like I don't even care if it's going to work, I can't take more change." Andreessen believes that the major barrier to change is sociological: people can embrace only so many new ideas at once. "O.K., Google, O.K., Twit-

ter—but Airbnb? People staying in each other's houses without there being a lot of axe murders?"

A16z passed on Airbnb's A round in 2009. Reid Hoffman, the Greylock V.C., who led that round, and who is a friend of Andreessen's, said, "Once something like Airbnb gets going, Marc can get a very good sense of it, of the economic system—but he's not necessarily as good at the psychology of why it would get going in the first place."

Brian Chesky, Airbnb's co-founder and C.E.O., told me, "In 2011, when we were starting to get some traction, Marc and Ben did a one-eighty and were very humble. Marc said he now saw it through the lens of eBay: buying stuff from strangers." A16z led Airbnb's B round. Soon afterward, the company was battered by headlines about renters who trashed a San Francisco home. It wasn't axe murders, but, Chesky said, "It was a P.R. nightmare. We had just expanded from being ten people living in a three-bedroom apartment and we had no idea how to be a billion-dollar company. Marc came to our office at midnight and read the letter I'd written to our community about the Airbnb Guarantee, and the two changes he made changed the company forever. I'd said we guarantee five thousand dollars for property damage, and he added a zero, which seemed crazy." Andreessen also added the proviso that claimants would have to file a police report, which he correctly believed would discourage scam artists. "And he told me to add my personal e-mail address. He gave us permission to be bold."

In venture, it's not batting average that matters; it's slugging average. Boldness is all. When Google Glass appeared, a16z joined a collective to seek out investments, and Andreessen declared that, without the face shield, "people are going to find they feel, basically, naked and lonely." Google withdrew the product in January. But, he would argue, so what? His thesis is that such a16z failures as Fab and Rockmelt and Digg and Kno are not merely a tolerable by-product of the risk algorithm but a vital indicator of it. It's fine to have a lousy record of predicting the future, most of the time, as long as when you're right you're really right. Between 2004 and 2013, a mere 0.4 per cent of all venture investments returned at least 50x. The real mistakes aren't the errors of commission, the companies that crash—all you can lose is your investment—but those of omission. There were good reasons that a16z passed on buying twelve per cent of Uber in 2011, including a deadline of just hours to make a decision. But the firm missed a profit, on paper, of more than three billion dollars.

The beauty of betting on risky technologies is that you're sometimes proved right, eventually—perhaps we'll all feel naked without Google Glass 3.0. When reverses occur, Andreessen tends to believe that he wasn't wrong so much as overly pre-

scient. Yet, while he professes intellectual comfort with being wrong, he never mentions Ning, a social-networking company that he co-founded in 2004, because, as he conceded when I asked about the elision, "It didn't do great." And he can be touchy about criticism. At one Q. & A. I attended, when the interviewer read him a snarky quote from Sam Biddle, a writer who worked for the gossip site Valleywag, Andreessen made a doobie-smoking gesture and plunged an imaginary needle into his vein to suggest the quality of Biddle's thinking. Being the public face of venture means that you can be challenged on multiple fronts: even as you philosophize about ushering in a new age of democracy, you also have to make money for your L.P.s. And, while the ideal startup advances both goals, most, in truth, advance neither. The V.C. Bryce Roberts told me, "It's an ego game, where you want to believe you're changing the world. But how can you write a check to Fab and believe that giving people discounted tchotchkes is changing the world?"

In 1999, Andreessen and Horowitz started Loudcloud, an early cloud-computing service that booked thirty-seven million dollars in contracts in its first nine months. Andreessen, meanwhile, was becoming Marc 2.0. He shed thirty pounds, started wearing Ermenegildo Zegna suits, and traded in his red Mustang for a white Mercedes. "Marc 1.0 was Jim Clark," Andreessen told me, referring to his impulsive co-founder. "Marc 2.0 was trying to get as polished as possible, more socialized. And Marc 3.0 is a combo. The goal is not to be elegant but to be blunt enough that there's no confusion. I learned the skills from reading all of Caro's L.B.J. books."

The dot-com crash hit Loudcloud hard, and, in 2002, it pivoted to become a software company with a new name: Opsware. In 2007, after years of slogging, Andreessen and Horowitz sold the company for $1.6 billion. Andreessen says that the tech crash scarred him: "The overwhelming message to our generation in the early nineties was 'You're dirty, you're all about grunge—you guys are fucking losers!' Then the tech boom hit, and it was 'We are going to do amazing things!' And then the roof caved in, and the wisdom was that the Internet was a mirage. I one hundred per cent believed that, because the rejection was so personal—both what everybody thought of me and what I thought of myself. I was not depressed, but I was growly. In retrospect," he concluded, "we were five or six years too early."

Peter Thiel, who is four years older than Andreessen, observed that "the late nineties, for Gen Xers in Silicon Valley, was an experience as powerful as the late sixties was for the younger boomers. The sixties was a transformative moment that got short-circuited by Nixon, and, for Marc, the nineties—when Netscape was iconic, and he was deeply living the belief that technology was going to inspire liberalization everywhere—was short-circuited by the super-powerful bust and return of the old economy. But Marc is very tenacious."

Andreessen said he learned that, while technology improves steadily, "psychologically there's no middle ground—the plane is always headed straight up or straight down." Recognizing that he was a poor manager, and needing to buffer those emotional and financial swings, Andreessen saw that the obvious next move was a portfolio of investments. In 2003, he and Horowitz began angel investing, separately and then together; they put ten million dollars into fifty companies, including Facebook, Twitter, and LinkedIn. Then Andreessen began pushing to start a venture firm. "I always thought the entire venture thing was incredibly cool," he told me. "Going to Kleiner Perkins"—the firm that funded Netscape—"with the high ceilings, the markers on the wall of all the great companies they'd I.P.O.'d, Larry Ellison walking through, and, at 11 a.m., the biggest buffet you've ever seen, at a time when I was eating at Subway? It was the closest thing to a cathedral for nerds." Mark Zuckerberg told me, "When Marc started Andreessen Horowitz, I asked him why he didn't start another company instead, and he said, 'It would be like going back to kindergarten.' "

A16z was designed not merely to succeed but also to deliver payback: it would right the wrongs that Andreessen and Horowitz had suffered as entrepreneurs. Most of those, in their telling, came from Benchmark Capital, the firm that funded Loudcloud, and recently led the A rounds of Uber and Snapchat—a five-partner boutique with no back-office specialists to provide the services they'd craved. "We were always the anti-Benchmark," Horowitz told me. "Our design was to not do what they did." Horowitz is still mad that one Benchmark partner asked him, in front of his co-founders, "When are you going to get a real C.E.O.?" And that Benchmark's best-known V.C., the six-feet-eight Bill Gurley, another outspoken giant with a large Twitter following, advised Horowitz to cut Andreessen and his six-million-dollar investment out of the company. Andreessen said, "I can't stand him. If you've seen 'Seinfeld,' Bill Gurley is my Newman"—Jerry's bête noire.

A16z's services model made a strong impression on Sand Hill Road. "Andreessen caused us to up our game on the marketing side," Sequoia's Doug Leone told me. "Younger founders pay attention to media, and we don't want to be de-positioned." Sequoia hired an in-house publicist and two new marketing specialists to complement the four it had, and most top firms made similar moves, even if they privately believed that a16z's services were simply a marketing tool. Todd McKinnon, the C.E.O. of Okta, said, "Every firm we talk to now is 'Hey, we're doing all this recruiting, and we'll introduce you to big customers.' It's become the table stakes."

Benchmark, by contrast, took down its Web site. "It's like watching Coke and Pepsi go head to head," one C.E.O. told me. Bill Gurley declined my requests for comment,

but he has publicly bemoaned all the money that firms such as a16z are pumping into the system at a time when he and many other V.C.s worry that the tech sector is experiencing another bubble. So many investors from outside the Valley want in on the startup world that valuations have been soaring: last year, thirty-eight U.S. start-ups received billion-dollar valuations, twenty-three more than in 2013. Many V.C.s have told their companies to raise as much money as possible now, to have a buffer against a crash.

Benchmark's funds top out at four hundred million dollars and are reserved for early-round investing: the original, artisanal venture model. A16z raised $1.5 billion each for its third and fourth funds, in 2012 and 2014, with much of the money earmarked for later, costlier growth rounds, whose returns tend to be capped at 5x. Andreessen argues that startups now wait longer and raise more capital before going public, and a16z wants to be in those conversations, too. He also says that larger funds will allow the firm to provide even more of the services that its entrepreneurs crave. But, in the Valley, increasing your fund size so dramatically is customarily seen as "smoking your own exhaust," or, among those with a classical turn of mind, hubris. "Venture doesn't scale," Diane Mulcahy, the L.P. and venture critic, said. "Raising and investing more doesn't increase the number of billion-dollar companies, and offering services to entrepreneurs won't help the firm generate returns. It's like a Saks Fifth Avenue that gives everybody a free iPhone. Are they going to attract everybody and see everybody? Yes. Are they going to make money? Not for long."

When I pressed Andreessen on a16z's fund size, he said that even if the basic assumptions haven't changed—even if only fifteen companies a year reach a hundred million dollars in revenue—those companies generate more money now. And, he said, "I'd bet the number of companies that reach that revenue is going up." With a playful smile, he referred to Gurley: "If there's no profit opportunity beyond the first four hundred million, Bill's making the case that everyone who follows Benchmark in a later investment round is a moron. I wouldn't say that."

One morning, as I sat down to breakfast with Andreessen, a rival V.C. sent me a long e-mail about a16z's holdings. The V.C. estimated that because Andreessen's firm had taken so many growth positions, its average ownership stake was roughly 7.5 per cent (it's eight per cent), which meant that to get 5x to 10x across its four funds "you would need your aggregate portfolio to be worth $240-$480B!" You would, in other words, need to invest in every Facebook and Uber that came along. When I started to check the math with Andreessen, he made a jerking-off motion and said "Blah-blah-blah. We have all the models—we're elephant hunting, going after big game!"

In addition to assuaging various slights from V.C.s, Andreessen is attempting to assuage the wound of the 2000 crash, by maintaining that it was an isolated event. "The argument in favor of concern is cyclical," he told me—busts follow booms. "The counterargument is that stuff works now. In 2000, you had fifty million people on the Internet, and the number of smartphones was zero. Today, you have three billion Internet users and two billion smartphones. It's Pong versus Nintendo. It's Carlota Perez's argument that technology is adopted on an S curve: the installation phase, the crash—because the technology isn't ready yet—and then the deployment phase, when technology gets adopted by everyone and the real money gets made." So the 2000 tech crash prefigured not the next crash but a sustained boom. And Andreessen's portfolio, like the entire Sand Hill Road enterprise, wasn't so much overpriced as underappreciated.

Still, he recently tweeted that startups were spending too much. When the market turns, he wrote, "nobody will want to buy your cash-incinerating startup. There will be no Plan B. vaporize." And, come to think of it, maybe it wasn't prudent to raise too much, either. In one pitch meeting where a portfolio company sought a billion-dollar growth round, Andreessen raised his arms overhead and made an explosive sound to warn of what can happen when your valuation vastly exceeds your revenues: "Thanks for playing—game over!" The company went on to secure its round, with only a token contribution from a16z. Andreessen later said that, as in an increasing number of deals, growth investors had paid one round ahead of progress—paid in other words, for the results they hoped to see in the following round. Though the company's lofty valuation buoyed a16z's portfolio, his body language suggested that buying at such valuations was maybe not smart—"but, as long as they're sophisticated investors, it's not our job to moralize on whether they're overpaying."

Another way of framing the growth-funding question, Peter Thiel suggests, is that Andreessen may not be as suited to making early, counterintuitive investments as he is—a point that Andreessen concedes: "Peter is just smarter than I am, and in a lateral way." But, Thiel says, Andreessen is well positioned, because of his broad knowledge and flexible mind-set, to respond to incremental changes in an array of fields. And that, he argues, is what the times reward: "While Twitter is a lesser innovation than flying cars, it's a much more valuable business. We live in a financial age, not a technological age."

In December, Apoorva Mehta, the founder of a grocery-delivery app called Instacart, came to a16z to ask it to fill out his C round. The firm had led Mehta's B round with an investment of twenty-seven million dollars, but he reminded the team anyway that Instacart "is quite a magical experience." Then he invoked a few sharing-econ-

omy shibboleths, including “we don’t have any infrastructure,” “mobile-powered independent contractors,” and “machine-learning-based fulfillment engine.” In two years, Mehta had set up in fifteen cities, signed up many of the independent grocery chains, including Whole Foods, and showed profitability in a number of stores. And it was a defensible network, because he installed refrigerated lockers in the stores. At the same time, because Mehta had recently changed his model, Instacart was losing money on each delivery, and that amount was growing as he rapidly expanded into new markets.

Andreessen applied a disinfecting wipe and said, “Let me ask you a question I know the answer to. In 1999, there was no more flaming debacle of a business than grocery delivery online. You were probably twelve at the time of Webvan?”

“Thirteen,” Mehta said.

“So why now?”

“The main reason is you have access to labor through smartphones. It’s the same reason Uber and Lyft exist now.”

Andreessen nodded with satisfaction: “You can orchestrate the entire supply chain through your phone.” Webvan was what he called a “ghost story”—a cautionary tale that still frightened investors. But Instacart proved that even haunted houses could be rehabilitated.

Another partner asked about competitors, including Uber, TaskRabbit, Amazon Fresh, and Fresh Direct. “The other, older models can’t do instant delivery,” Mehta replied. “And the newer ones don’t have anywhere near our coverage and range of data in groceries. So if you want slower delivery and smaller selection, go with them.” Andreessen smiled, savoring the contempt.

At the deal review, Jeff Jordan, who sits on Instacart’s board, praised Mehta’s progress, while noting concerns about unit economics—how he’d get to profitability on each delivery. Referring to the venture community’s enthusiasm for the round, Jordan went on, “This is an ‘I missed Uber, I don’t want to miss the next one’ climate.” Balancing everything, he recommended that the firm put in ten million dollars.

Horowitz argued for a bigger investment. Mehta’s moat against competitors “is really fucking deep—he already has Whole Foods, monster of monsters. It’s the biggest market of all time, incredibly huge.”

After other partners argued that the valuation seemed high, Andreessen looked at Horowitz: "Ben, I think you're making an even more provocative point than people understand. It sounds like you're saying this could be an Uber for real."

"I think so," Horowitz said. "What makes unit economics really scary is if you're in a competitive market. He's in a monopoly."

Andreessen said, "We could go to the well, and go in higher." He beckoned, coaxingly. Horowitz thought it over, then said, "I don't want to override Jeff." Andreessen, too, seemed content to temper his enthusiasm and to share the round with other firms. (Mehta eventually raised two hundred and twenty million dollars on a valuation of two billion.) He'd like to make twenty times the investments the firm does, but every opportunity comes with an opportunity cost, and even $1.5 billion doesn't last forever.

Andrew Golden, the chief investment officer for Princeton University, an L.P. in a16z's last three funds, told me that, when the firm started, "my worry was that Marc is such a big personality he wouldn't necessarily listen to someone who told him he was wearing fewer clothes than he thought. But now my working hypothesis is that Marc is smart enough to know that he'll do better if he doesn't try to win every argument—if he doesn't try to go undefeated."

In March, Andreessen and his wife announced the birth of their son, who'd been carried to term by a gestational surrogate. They named him John, for Laura's father. "I feel fantastic!" Andreessen told me. "He'll come of age in a world where ten or a hundred times more people will be able to contribute in science and medicine and the arts, a more peaceful and prosperous world." He added, tongue in cheek, "I'm going to teach him how to take over that world!"

Andreessen often remarks that, in the blue-collar milieu he came from, no parent wants his or her child to stay blue-collar. His own circumstances have changed dramatically—he is now a paper billionaire, though he argues that his net worth depends on how you value a16z—so I told him it seemed paradoxical that some of his other babies, such as Instacart and Lyft, make their profits off blue-collar drivers and pickers who must freelance without a safety net to make ends meet. Unsurprisingly, he strongly disagreed: "Maybe there's an alternate way of living, a free-form life where you press the button and get work when you want to."

One afternoon, as we sat at his baronial dining table, he made an agonized but sincere effort to discuss his blue-collar childhood without mentioning his nuclear family. "I really identified with Charles Schulz in the David Michaelis biography of him,

‘Schulz and Peanuts,’ ” he said. I was struck by the parallels between Andreessen and both “Peanuts”—in which Charlie Brown has a massive bald head and the parents are kept offstage—and its creator. Charles Schulz, who grew up in Minnesota, was socially awkward, hated being embraced, and loathed his mother’s Norwegian relatives, a farming family. Andreessen went on, “Ninety-six per cent of the people who grow up like he and I did, in the Midwest, just stay there, but the ones who leave”—the cartoonist, too, moved to California—“become intensely interested in the future. In Schulz’s last ten years, he really focussed on Rerun, Linus’s younger brother—the youngest and most optimistic character.”

I told Andreessen that this seemed like a tendentious reading of Rerun, a bland character whose two most famous lines are “I’ll drink to that” and “My brother is the only one in the family with a blanket, and I don’t want to end up like him.” Taken aback, he explained, “He’s the youngest, he’s the newest, he has the most life in front of him.” Andreessen, as he saw himself, was both an immigrant to the land of opportunity, like the entrepreneurs he preferred to fund, and someone whose childhood was merely an installation phase. He told me, “It wasn’t that I felt misunderstood or badly treated; it was that I was so completely different. I wasn’t seeking understanding. I wasn’t indexing myself against the people around me.”

Andreessen reminded me—in his formidable achievements and manner, his thickly armored sensitivities and yearnings—of Rilke’s remark “Perhaps everything that frightens us is, in its deepest essence, something helpless that wants our love.” When I told him so, he stared back in absolute horror.

Last year, a programmer named Alex Payne wrote an open letter to Andreessen in which he observed, “People are scared of so much wealth and control being in so few hands. Consequently, wherever you and other gatekeepers of capital direct your attention—towards robots, 3D printers, biotech, whatever—you’re going to detect a fearful response as people scramble to determine the impact of your decisions and whims,” which only compound “lingering structural unemployment and an accumulation of capital at the top of the economic pyramid.”

Payne addressed his thoughts to Andreessen because Andreessen represents the Valley—both in its soaring vision and in its tendency to treat people as a fungible mass. But Andreessen waved away the criticisms as the ravings of “a self-hating software engineer.” When I persisted, he said, “Ordinary people love the iPhone, Facebook, Google Search, Airbnb, and Lyft. It’s only the intellectuals who worry.” He raised counter-arguments, then dismissed them: technology would solve any environmental crisis hastened by an expanding economy, and as for the notion that, as he said, “ ‘You American imperialist asshole, not everyone wants all that technolo-

gy'—well, bullshit! Go to a Chinese village and ask them." Technology gives us superpowers, makes us smarter, more powerful, happier. "Would the world be a better place if there were fifty Silicon Valleys?" he said. "Obviously, yes. Over the past thirty years, the level of income throughout the developing world is rising, the number of people in poverty is shrinking, health outcomes are improving, birth rates are falling. And it'll be even better in ten years. Pessimism always sounds more sophisticated than optimism—it's the Eden-collapse myth over and over again—and then you look at G.D.P. per capita worldwide, and it's up and to the right. If this is collapse, let's have more of it!"

Global unemployment is rising, too—this seems to be the first industrial revolution that wipes out more jobs than it creates. One 2013 paper argues that forty-seven per cent of all American jobs are destined to be automated. Andreessen argues that his firm's entire portfolio is creating jobs, and that such companies as Udacity (which offers low-cost, online "nanodegrees" in programming) and Honor (which aims to provide better and better-paid in-home care for the elderly) bring us closer to a future in which everyone will either be doing more interesting work or be kicking back and painting sunsets. But when I brought up the raft of data suggesting that intra-country inequality is in fact increasing, even as it decreases when averaged across the globe—America's wealth gap is the widest it's been since the government began measuring it—Andreessen rerouted the conversation, saying that such gaps were "a skills problem," and that as robots ate the old, boring jobs humanity should simply retool. "My response to Larry Summers, when he says that people are like horses, they have only their manual labor to offer"—he threw up his hands. "That is such a dark and dim and dystopian view of humanity I can hardly stand it!"

One challenge for Andreessen is whether venture itself has a skills problem. If software is truly eating the world, wouldn't venture capital be on the menu? The AngelList platform now allows investors to fund startups online. Its co-founder Naval Ravikant said that "future companies will require more two-hundred-thousand-dollar checks and way fewer guys on Sand Hill Road." Jeff Fagnan, of Atlas Venture, which is the largest investor in AngelList, said, "Software is already squeezing out other intermediaries—travel agents, financial advisers—and, at the end of the day, V.C.s are intermediaries. We're all just selling cash."

Andreessen sometimes wonders if Ravikant is onto something. He's asked Horowitz, "What if we're the most evolved dinosaur, and Naval is a bird?" Already, more than half the tech companies that reached a billion-dollar valuation in the past decade were based outside Silicon Valley. And as Andreessen himself wrote in 2007, before he became a V.C., "Odds are, nothing your V.C. does, no matter how helpful or well-intentioned, is going to tip the balance between success and failure."

He still believes that—but he also thinks that a16z can cut a company's time to success in half, and time is money. He also believes that venture will maintain its incumbency because computers can't yet introduce you to just the right engineer or chief information officer at eBay, and machines can't yet come to your office at midnight to future-proof your letter to perturbed customers. Indeed, venture is one of the most human businesses going. Only human beings could have created such a supercollider of contradictions: a font of innovation that pools around conformity; a freedom train that speeds toward monopoly; a promoter of transparency that shrouds its own dealings; a guild that's dedicated to flattening hierarchies, and that rewards its leaders with imperial power.

Naturally, Andreessen had to weigh the counterargument, and consider whether he added any value at all. One Sunday afternoon, as he sat alone at the head of a16z's conference table, he said, "Chris Dixon argues that we're in the magical-products business—that we fool ourselves into thinking we're building companies, but it doesn't matter if we don't have the magical products." And magic could not be summoned, only prepared for. "Over twenty years," he continued, "our returns are going to come down to two or three or four investments, and the rest of this"—his gesture took in the building full of art, the devotions of more than a hundred eager souls, even the faux-Moorish rooftops of his competitors down the road—"is the cost of getting the chance at those investments. There's a sense in which all of this is math—you just don't know which Tuesday Mark Zuckerberg is going to walk in."

Yet math was no help with mass psychology. "Even if we could do perfect analysis, we just can't know the future," he said. "What if Google Ventures had access to all Google searches—could you predict hit products? Or perfect access to all of people's conversations or purchases? You still wouldn't know what's going to happen. How is psychohistory going?" he went on, referring to Isaac Asimov's invention, in his "Foundation" novels, of a statistical field that could predict the behavior of civilizations. "Not very fucking good at all! Which, by the way, is part of what makes this job really fun. It's a people business. If we could revise the industry completely, we'd just dump all the business plans and focus on people—the twenty-three-year-old Mark Zuckerberg, Bill Gates, Steve Jobs."

He acknowledged, though, that his optimism dims once human beings—with their illogic, hidden agendas, and sheer bugginess—enter the equation. "We're imperfect people pursuing perfect ideas, and there's tremendous frustration in the gap," he said. "Writing code, one or two people, that's the Platonic ideal. But when you want to impact the world you need one hundred people, then one thousand, then ten thousand—and people have all these people issues." He examined the problem

in silence. “A world of just computers wouldn’t work,” he concluded wistfully. “But a world of just people could certainly be improved.”

Cooley–Tukey FFT algorithm

Wikipedia

The Cooley–Tukey algorithm, named after J.W. Cooley and John Tukey, is the most common fast Fourier transform (FFT) algorithm. It re-expresses the discrete Fourier transform (DFT) of an arbitrary composite size N = N1N2 in terms of smaller DFTs of sizes N1 and N2, recursively, in order to reduce the computation time to O(N log N) for highly composite N (smooth numbers). Because of the algorithm's importance, specific variants and implementation styles have become known by their own names, as described below.

Because the Cooley-Tukey algorithm breaks the DFT into smaller DFTs, it can be combined arbitrarily with any other algorithm for the DFT. For example, Rader's or Bluestein's algorithm can be used to handle large prime factors that cannot be decomposed by Cooley–Tukey, or the prime-factor algorithm can be exploited for greater efficiency in separating out relatively prime factors.

The algorithm, along with its recursive application, was invented by Carl Friedrich Gauss. Cooley and Tukey independently rediscovered and popularized it 160 years later.

See also the fast Fourier transform for information on other FFT algorithms, specializations for real and/or symmetric data, and accuracy in the face of finite floating-point precision.

History

This algorithm, including its recursive application, was invented around 1805 by Carl Friedrich Gauss, who used it to interpolate the trajectories of the asteroids Pallas and Juno, but his work was not widely recognized (being published only posthumously and in neo-Latin).[1][2] Gauss did not analyze the asymptotic computational time, however. Various limited forms were also rediscovered several times throughout the 19th and early 20th centuries.[2] FFTs became popular after James Cooley of IBM and John Tukey of Princeton published a paper in 1965 reinventing the algorithm and describing how to perform it conveniently on a computer.[3]

Tukey reportedly came up with the idea during a meeting of a US presidential advisory committee discussing ways to detect nuclear-weapon tests in the Soviet Union.[4][5] Another participant at that meeting, Richard Garwin of IBM, recognized the potential of the method and put Tukey in touch with Cooley, who implemented it

for a different (and less-classified) problem: analyzing 3d crystallographic data (see also: multidimensional FFTs). Cooley and Tukey subsequently published their joint paper, and wide adoption quickly followed.

The fact that Gauss had described the same algorithm (albeit without analyzing its asymptotic cost) was not realized until several years after Cooley and Tukey's 1965 paper.[2] Their paper cited as inspiration only work by I. J. Good on what is now called the prime-factor FFT algorithm (PFA);[3] although Good's algorithm was initially mistakenly thought to be equivalent to the Cooley–Tukey algorithm, it was quickly realized that PFA is a quite different algorithm (only working for sizes that have relatively prime factors and relying on the Chinese Remainder Theorem, unlike the support for any composite size in Cooley–Tukey).[6]

The radix-2 DIT case

A radix-2 decimation-in-time (DIT) FFT is the simplest and most common form of the Cooley–Tukey algorithm, although highly optimized Cooley–Tukey implementations typically use other forms of the algorithm as described below. Radix-2 DIT divides a DFT of size N into two interleaved DFTs (hence the name "radix-2") of size N/2 with each recursive stage.

The discrete Fourier transform (DFT) is defined by the formula:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk},$$

where k is an integer ranging from 0 to N-1.

Radix-2 DIT first computes the DFTs of the even-indexed inputs $(x_{2m} = x_0, x_2, \ldots, x_{N-2})$ and of the odd-indexed inputs $(x_{2m+1} = x_1, x_3, \ldots, x_{N-1})$, and then combines those two results to produce the DFT of the whole sequence. This idea can then be performed recursively to reduce the overall runtime to O(N log N). This simplified form assumes that N is a power of two; since the number of sample points N can usually be chosen freely by the application, this is often not an important restriction.

The Radix-2 DIT algorithm rearranges the DFT of the function x_n into two parts: a sum over the even-numbered indices $n = 2m$ and a sum over the odd-numbered indices $n = 2m + 1$:

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N}(2m)k} + \sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N}(2m+1)k}$$

One can factor a common multiplier $e^{-\frac{2\pi i}{N}k}$ out of the second sum, as shown in the equation below. It is then clear that the two sums are the DFT of the even-indexed part x_{2m} and the DFT of odd-indexed part x_{2m+1} of the function x_n. Denote the DFT of the Even-indexed inputs x_{2m} by E_k and the DFT of the Odd-indexed inputs x_{2m+1} by O_k and we obtain:

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2}mk}}_{\text{DFT of even-indexed part of } x_m} + e^{-\frac{2\pi i}{N}k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2}mk}}_{\text{DFT of odd-indexed part of } x_m} = E_k + e^{-\frac{2\pi i}{N}k} O_k.$$

Thanks to the periodicity of the DFT, we know that

$$E_{k+\frac{N}{2}} = E_k$$

and

$$O_{k+\frac{N}{2}} = O_k$$

. Therefore, we can rewrite the above equation as

$$X_k = \begin{cases} E_k + e^{-\frac{2\pi i}{N}k} O_k & \text{for } 0 \le k < N/2 \\ E_{k-N/2} + e^{-\frac{2\pi i}{N}k} O_{k-N/2} & \text{for } N/2 \le k < N. \end{cases}$$

We also know that the twiddle factor $e^{-2\pi ik/N}$ obeys the following relation:

$$\begin{aligned} e^{\frac{-2\pi i}{N}(k+N/2)} &= e^{\frac{-2\pi ik}{N} - \pi i} \\ &= e^{-\pi i} e^{\frac{-2\pi ik}{N}} \\ &= -e^{\frac{-2\pi ik}{N}} \end{aligned}$$

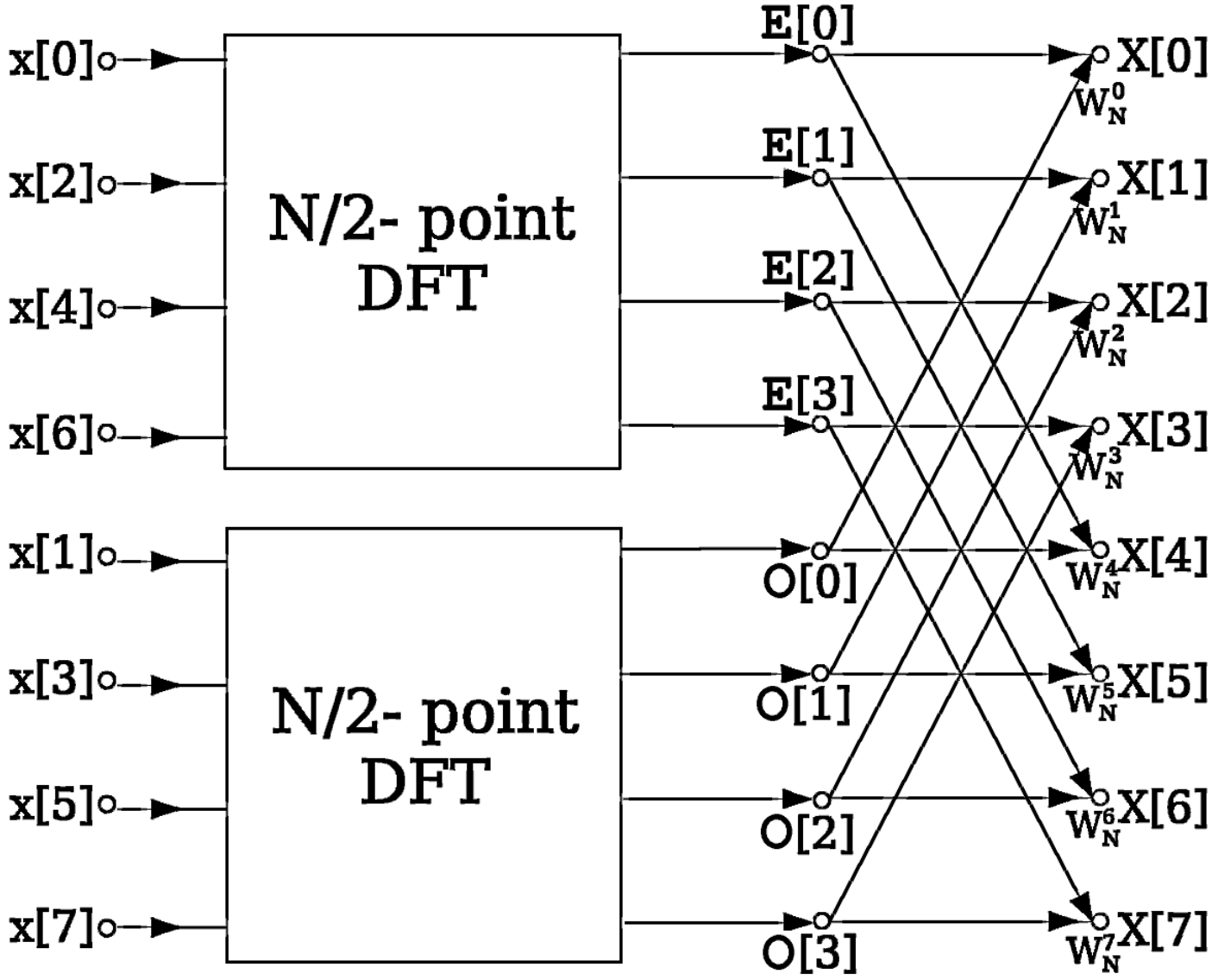
This allows us to cut the number of "twiddle factor" calculations in half also. For $0 \le k < \frac{N}{2}$, we have

$$\begin{aligned} X_k &= E_k + e^{-\frac{2\pi i}{N}k} O_k \\ X_{k+\frac{N}{2}} &= E_k - e^{-\frac{2\pi i}{N}k} O_k \end{aligned}$$

This result, expressing the DFT of length N recursively in terms of two DFTs of size N/2, is the core of the radix-2 DIT fast Fourier transform. The algorithm gains its speed by re-using the results of intermediate computations to compute multiple DFT outputs. Note that final outputs are obtained by a +/– combination of E_k and $O_k \exp(-2\pi ik/N)$, which is simply a size-2 DFT (sometimes called a butterfly in this context); when this is generalized to larger radices below, the size-2 DFT is replaced by a larger DFT (which itself can be evaluated with an FFT).

This process is an example of the general technique of divide and conquer algorithms; in many traditional implementations, however, the explicit recursion is avoided, and instead one traverses the computational tree in breadth-first fashion.

The above re-expression of a size-N DFT as two size-N/2 DFTs is sometimes called the Danielson–Lanczos lemma, since the identity was noted by those two authors in 1942[7] (influenced by Runge's 1903 work[2]). They applied their lemma in a "backwards" recursive fashion, repeatedly doubling the DFT size until the transform spectrum converged (although they apparently didn't realize the linearithmic [i.e., order N log N] asymptotic complexity they had achieved). The Danielson–Lanczos work predated widespread availability of computers and required hand calculation (possibly with mechanical aids such as adding machines); they reported a computation time of 140 minutes for a size-64 DFT operating on real inputs to 3–5 significant digits. Cooley and Tukey's 1965 paper reported a running time of 0.02 minutes for a size-2048 complex DFT on an IBM 7094 (probably in 36-bit single precision, ~8 digits).[3] Rescaling the time by the number of operations, this corresponds roughly to a speedup factor of around 800,000. (To put the time for the hand calculation in perspective, 140 minutes for size 64 corresponds to an average of at most 16 seconds per floating-point operation, around 20% of which are multiplications.)



Pseudocode

In pseudocode, the below procedure could be written:[8]

```
X0,...,N−1 ← ditfft2(x, N, s):                  # DFT of (x0, xs, x2s, ..., x(N−1)s):
    if N = 1 then
        X0 ← x0                                 # trivial size-1 DFT base case
    else
        X0,...,N/2−1 ← ditfft2(x, N/2, 2s)      # DFT of (x0, x2s, x4s, ...)
        XN/2,...,N−1 ← ditfft2(x+s, N/2, 2s)    # DFT of (xs, xs+2s, xs+4s, ...)
        for k = 0 to N/2−1                      # combine DFTs of two halves into full DFT:
            t ← Xk
            Xk ← t + exp(−2πi k/N) Xk+N/2
            Xk+N/2 ← t − exp(−2πi k/N) Xk+N/2
        endfor
    endif
```

Here, ditfft2(x,N,1), computes X=DFT(x) out-of-place by a radix-2 DIT FFT, where N is an integer power of 2 and s=1 is the stride of the input x array. x+s denotes the array starting with xs.

(The results are in the correct order in X and no further bit-reversal permutation is required; the often-mentioned necessity of a separate bit-reversal stage only arises for certain in-place algorithms, as described below.)

High-performance FFT implementations make many modifications to the implementation of such an algorithm compared to this simple pseudocode. For example, one can use a larger base case than N=1 to amortize the overhead of recursion, the twiddle factors $\exp[-2\pi ik/N]$ can be precomputed, and larger radices are often used for cache reasons; these and other optimizations together can improve the performance by an order of magnitude or more.[8] (In many textbook implementations the depth-first recursion is eliminated entirely in favor of a nonrecursive breadth-first approach, although depth-first recursion has been argued to have better memory locality.[8][9]) Several of these ideas are described in further detail below.

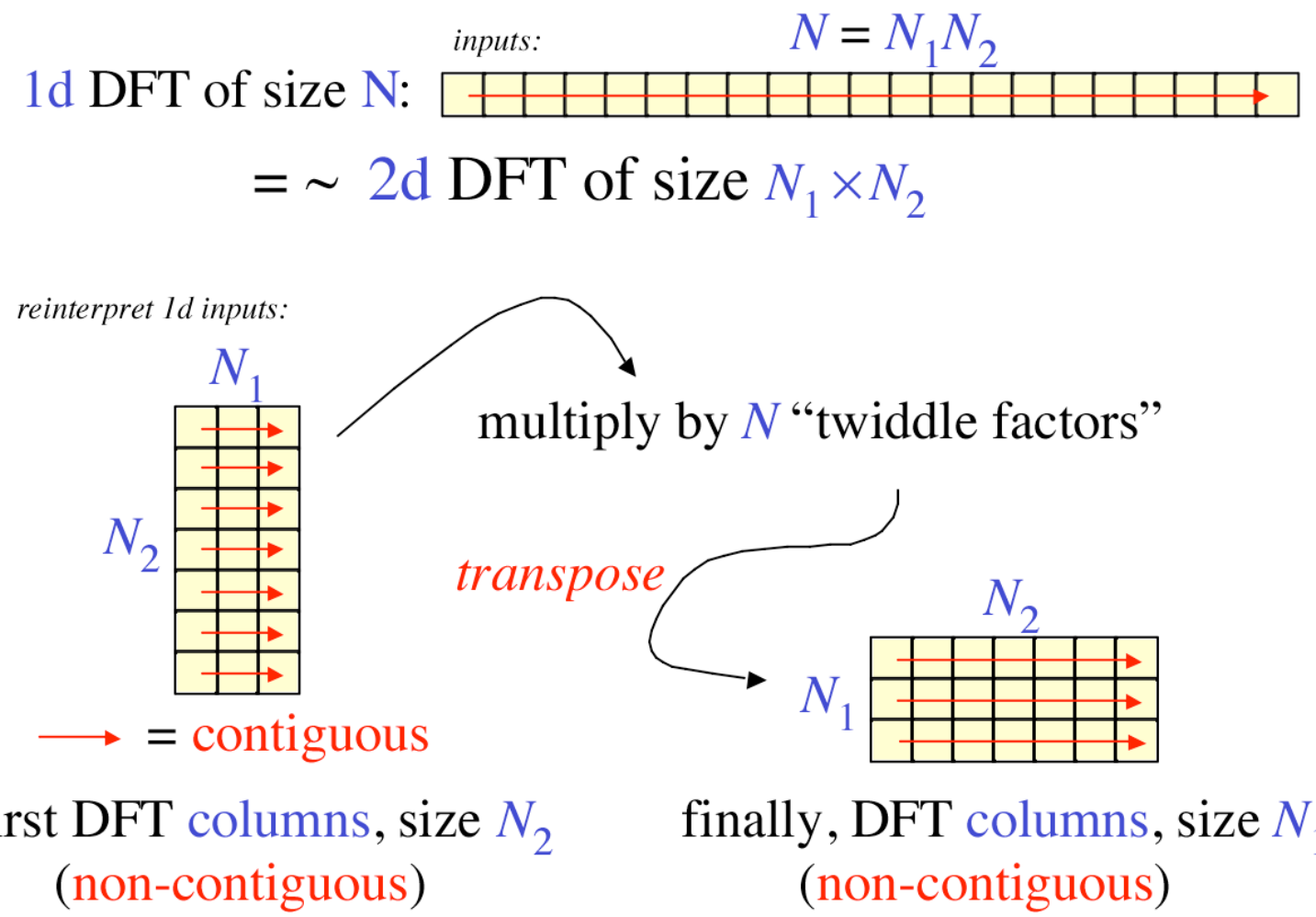
General factorizations

More generally, Cooley–Tukey algorithms recursively re-express a DFT of a composite size N = N1N2 as:[10]

Perform N1 DFTs of size N2.
Multiply by complex roots of unity called twiddle factors.
Perform N2 DFTs of size N1.

Typically, either N1 or N2 is a small factor (not necessarily prime), called the radix

(which can differ between stages of the recursion). If N1 is the radix, it is called a decimation in time (DIT) algorithm, whereas if N2 is the radix, it is decimation in frequency (DIF, also called the Sande-Tukey algorithm). The version presented above was a radix-2 DIT algorithm; in the final expression, the phase multiplying the odd transform is the twiddle factor, and the +/- combination (butterfly) of the even and odd transforms is a size-2 DFT. (The radix's small DFT is sometimes known as a butterfly, so-called because of the shape of the dataflow diagram for the radix-2 case.)



There are many other variations on the Cooley–Tukey algorithm. Mixed-radix implementations handle composite sizes with a variety of (typically small) factors in addition to two, usually (but not always) employing the O(N2) algorithm for the prime base cases of the recursion (it is also possible to employ an N log N algorithm for the prime base cases, such as Rader's or Bluestein's algorithm). Split radix merges radices 2 and 4, exploiting the fact that the first transform of radix 2 requires no twiddle factor, in order to achieve what was long the lowest known arithmetic operation count for power-of-two sizes,[10] although recent variations achieve an even lower count.[11][12] (On present-day computers, performance is determined more by cache and CPU pipeline considerations than by strict operation counts; well-optimized FFT implementations often employ larger radices and/or hard-coded base-case transforms of significant size.[13]) Another way of looking at the Cooley–Tukey algorithm is that it re-expresses a size N one-dimensional DFT as an N1 by N2 two-dimensional DFT (plus twiddles), where the output matrix is transposed. The net result of all of these transpositions, for a radix-2 algorithm, corresponds to a bit

reversal of the input (DIF) or output (DIT) indices. If, instead of using a small radix, one employs a radix of roughly √N and explicit input/output matrix transpositions, it is called a four-step algorithm (or six-step, depending on the number of transpositions), initially proposed to improve memory locality,[14][15] e.g. for cache optimization or out-of-core operation, and was later shown to be an optimal cache-oblivious algorithm.[16]

The general Cooley–Tukey factorization rewrites the indices k and n as $k = N_2 k_1 + k_2$ and $n = N_1 n_2 + n_1$, respectively, where the indices ka and na run from 0..Na-1 (for a of 1 or 2). That is, it re-indexes the input (n) and output (k) as N1 by N2 two-dimensional arrays in column-major and row-major order, respectively; the difference between these indexings is a transposition, as mentioned above. When this re-indexing is substituted into the DFT formula for nk, the $N_1 n_2 N_2 k_1$ cross term vanishes (its exponential is unity), and the remaining terms give

$$X_{N_2 k_1 + k_2} = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{N_1 n_2 + n_1} e^{-\frac{2\pi i}{N_1 N_2} \cdot (N_1 n_2 + n_1) \cdot (N_2 k_1 + k_2)}$$

where each inner sum is a DFT of size N2, each outer sum is a DFT of size N1, and the [...] bracketed term is the twiddle factor.

An arbitrary radix r (as well as mixed radices) can be employed, as was shown by both Cooley and Tukey[3] as well as Gauss (who gave examples of radix-3 and radix-6 steps).[2] Cooley and Tukey originally assumed that the radix butterfly required O(r2) work and hence reckoned the complexity for a radix r to be O(r2 N/r logrN) = O(N log2(N) r/log2r); from calculation of values of r/log2r for integer values of r from 2 to 12 the optimal radix is found to be 3 (the closest integer to e, which minimizes r/log2r).[3][17] This analysis was erroneous, however: the radix-butterfly is also a DFT and can be performed via an FFT algorithm in O(r log r) operations, hence the radix r actually cancels in the complexity O(r log(r) N/r logrN), and the optimal r is determined by more complicated considerations. In practice, quite large r (32 or 64) are important in order to effectively exploit e.g. the large number of processor registers on modern processors,[13] and even an unbounded radix r=√N also achieves O(N log N) complexity and has theoretical and practical advantages for large N as mentioned above.[14][15][16]

Data reordering, bit reversal, and in-place algorithms

Although the abstract Cooley–Tukey factorization of the DFT, above, applies in some form to all implementations of the algorithm, much greater diversity exists in the techniques for ordering and accessing the data at each stage of the FFT. Of

special interest is the problem of devising an in-place algorithm that overwrites its input with its output data using only O(1) auxiliary storage.

The most well-known reordering technique involves explicit bit reversal for in-place radix-2 algorithms. Bit reversal is the permutation where the data at an index n, written in binary with digits b4b3b2b1b0 (e.g. 5 digits for N=32 inputs), is transferred to the index with reversed digits b0b1b2b3b4 . Consider the last stage of a radix-2 DIT algorithm like the one presented above, where the output is written in-place over the input: when E_k and O_k are combined with a size-2 DFT, those two values are overwritten by the outputs. However, the two output values should go in the first and second halves of the output array, corresponding to the most significant bit b4 (for N=32); whereas the two inputs E_k and O_k are interleaved in the even and odd elements, corresponding to the least significant bit b0. Thus, in order to get the output in the correct place, b0 should take the place of b4 and the index becomes b0b4b3b2b1. And for next recursive stage, those 4 least significant bits will become b1b4b3b2, If you include all of the recursive stages of a radix-2 DIT algorithm, all the bits must be reversed and thus one must pre-process the input (or post-process the output) with a bit reversal to get in-order output. (If each size-N/2 subtransform is to operate on contiguous data, the DIT input is pre-processed by bit-reversal.) Correspondingly, if you perform all of the steps in reverse order, you obtain a radix-2 DIF algorithm with bit reversal in post-processing (or pre-processing, respectively). Alternatively, some applications (such as convolution) work equally well on bit-reversed data, so one can perform forward transforms, processing, and then inverse transforms all without bit reversal to produce final results in the natural order.

Many FFT users, however, prefer natural-order outputs, and a separate, explicit bit-reversal stage can have a non-negligible impact on the computation time,[13] even though bit reversal can be done in O(N) time and has been the subject of much research.[18][19][20] Also, while the permutation is a bit reversal in the radix-2 case, it is more generally an arbitrary (mixed-base) digit reversal for the mixed-radix case, and the permutation algorithms become more complicated to implement. Moreover, it is desirable on many hardware architectures to re-order intermediate stages of the FFT algorithm so that they operate on consecutive (or at least more localized) data elements. To these ends, a number of alternative implementation schemes have been devised for the Cooley–Tukey algorithm that do not require separate bit reversal and/or involve additional permutations at intermediate stages.

The problem is greatly simplified if it is out-of-place: the output array is distinct from the input array or, equivalently, an equal-size auxiliary array is available. The Stockham auto-sort algorithm[21][22] performs every stage of the FFT out-of-place,

typically writing back and forth between two arrays, transposing one "digit" of the indices with each stage, and has been especially popular on SIMD architectures.[22] [23] Even greater potential SIMD advantages (more consecutive accesses) have been proposed for the Pease algorithm,[24] which also reorders out-of-place with each stage, but this method requires separate bit/digit reversal and O(N log N) storage. One can also directly apply the Cooley–Tukey factorization definition with explicit (depth-first) recursion and small radices, which produces natural-order out-of-place output with no separate permutation step (as in the pseudocode above) and can be argued to have cache-oblivious locality benefits on systems with hierarchical memory.[9][13][25]

A typical strategy for in-place algorithms without auxiliary storage and without separate digit-reversal passes involves small matrix transpositions (which swap individual pairs of digits) at intermediate stages, which can be combined with the radix butterflies to reduce the number of passes over the data.[13][26][27][28][29]

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